

i) Recollections on HH*, THH* for rings

2) HH and THH* for schemes

3) Computations

) Recorp on HH* THH* K a ciring, A e K-alq The enveloping algebra is $A := A \otimes_{K} A^{op}$ A_K-modules = A-bimodules The Hochschild cohondogy $HH_{k}^{h}(A,M) := E_{x} + h_{A_{k}^{e}}^{h}(A,M)$

Car compute HHX Via the bar complex. Thm $0 \rightarrow A \rightarrow A \rightarrow 0$ Higher HH have interpretations.

Examples Ka field, $\left(\frac{k[\epsilon]}{\epsilon^2} \rightarrow K\right) \in H_K^2(\kappa)$ = 0 $\mathbb{Z}/p^2 \to \mathbb{Z}/p$ is not Z/p-linear is not Z-split. detected by HH*. not ~7

THH²(A,M) in bijection mit is (all square-zero) Zextensions of A by M

if A is a ring, its topological enveloping algebra is $A_s^e := A \otimes A^{2P}$. Hs topological Hochschild cohomology is $THH^{n}(A,M) := E_{x} + \frac{1}{A_{s}} \left(A,M \right)$

Relative version R a commutative ring spectrum, A an R-algebra Can form A^e_R & THHR USUAL THH is THHS. when Kisa fieb, THHK ~ HHK Shn Shukla THHKA EXTAXA (A,A)

Sample computations $Q_{s}^{E} = Q \otimes_{s} Q \simeq Q$ so if A is a \mathbb{Q} -alg., THH*(A,M)~HH*(A,M) Bökstedt period:city: K a perfect field, char p $THH^{*}(k) \simeq K[u]$

2) HH* & THH* for schemes

How to extend the definition of THH* to schemes?

Convention All schemes are separated & noetherian over a

field K

Defining HH* X a scheme. I) define $HH_{K}^{*}(X)$ locally & glue

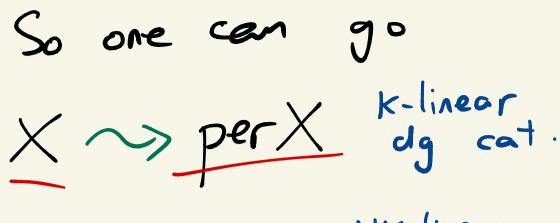
2) $HH_{x}^{*}(x) := E_{x}t_{X \times x}^{*}(\partial_{\Delta}, \partial_{\Delta})$

 $3HH_{K}^{*}(x) := HH_{K}^{*}(PerX)$

We choose 3) as our model for THHK.

On homology, done by Blumberg-Mandell.

Idea Just like one can take HH_K^* of a K-linear dg category, one can take THH* of a spectral category (= category enriched in) Spectra

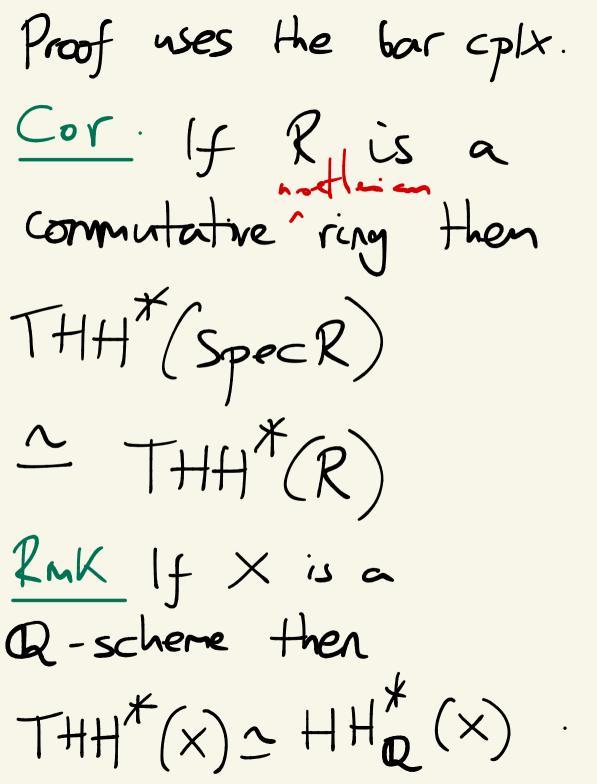


~ perX HK-linear spectral cat.

<u>per</u> <u>Spectral</u> cat.



Thm B. - Kaledin - Lowen (after Laven-Van den Bergh THHX satisfies strong invariance results for special categories. In particular one has THHX ~ THH perX - THH D CohX -THH*OQGLX <u>∼</u> . . .



3) Sample computations X smooth proper / a field K. Thm (BKL) Ja multiplicative, convergent upper half-plane spectral sequence with E² page $HH_{K}^{F}(X)\otimes_{K}THH^{I}(K)$ \implies THH^{P+9}(X)

In particular if K is perfect of characteristic p>0 we get $HH_{K}^{*}(X)[u] \Longrightarrow THH_{K}^{*}(X)$ This degenerates at E² when: •X a curve •X is P² and p>2 •X a K3 surface & p>2

e.g. $X = P_{Z/P}^{1}$. Then $THH^{*}X$ is

Z/p[a,b,c,u]/a²,b²,c² ab,bc,ac

RMK THHSX is always an Ez-algebra so THHX is graded - commutative.

Where to go now? · It's unclear in what sense we have THHX ~ {nonlinear JHHX ~ {deformations} J X when X is non-affine show up as an elemen an element of $T'HH^{2}(P_{Z/P}) \simeq \mathbb{Z}/P$

· Loose idea: THHX parameterises non-additive deformations of CohX

· Precise relationship between TH+1* & deformations of abelian categories is work in prograss.



for

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Aring M A-fim

 $THH^{*}(A, M)$

 $\sim 1/M^{*}(A,M)$