

# Algebraic Geometry and Singularities

## How to fold up space

Matt Booth

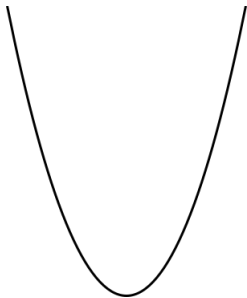
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Science Week 2023

# Algebraic geometry

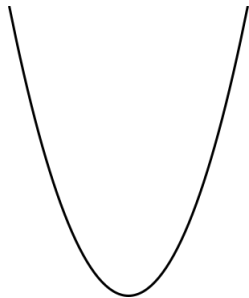
# Algebraic geometry

The geometry...



# Algebraic geometry

The geometry...

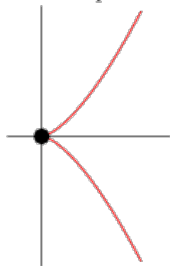


of algebraic equations

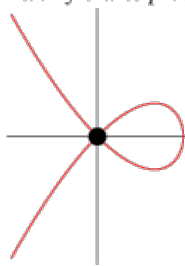
$$y = x^2$$

## More curves

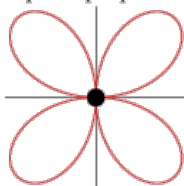
*cuspidal point*



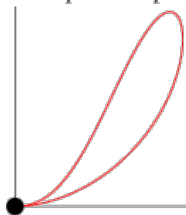
*ordinary double point*



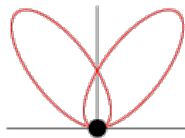
*quadruple point*



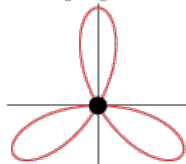
*ramphoid cusp*



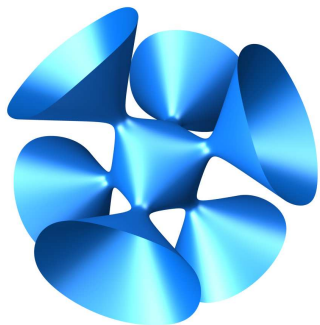
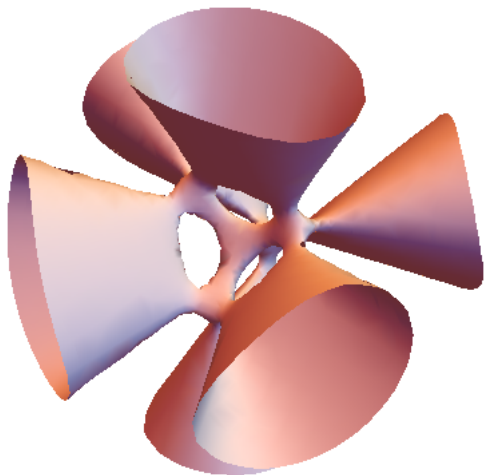
*tacnode*



*triple point*

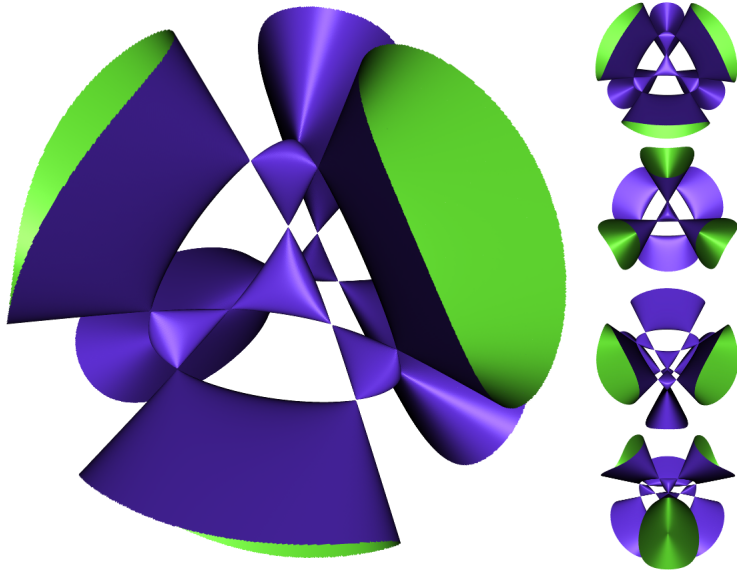


## K3 surfaces



$$1 + x^4 + y^4 + z^4 + a(x^2 + y^2 + z^2 + 1)^2 = 0, \quad a = -0.49$$

# a Kummer surface

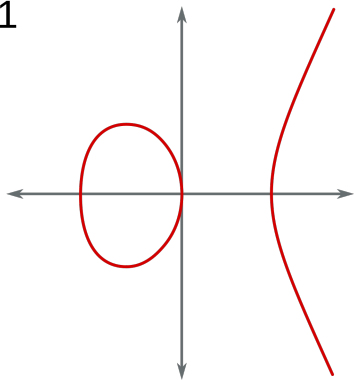


## Application: Elliptic curve cryptography



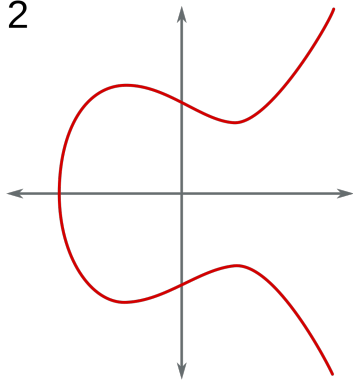
## Application: Elliptic curve cryptography

1



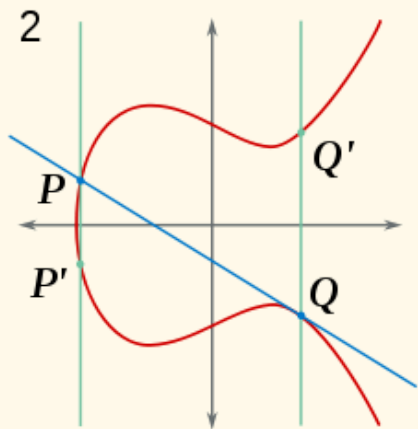
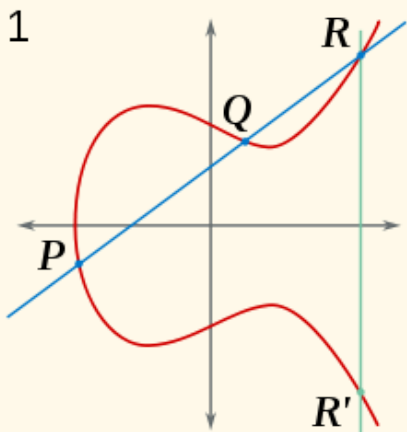
$$y^2 = x^3 - x$$

2



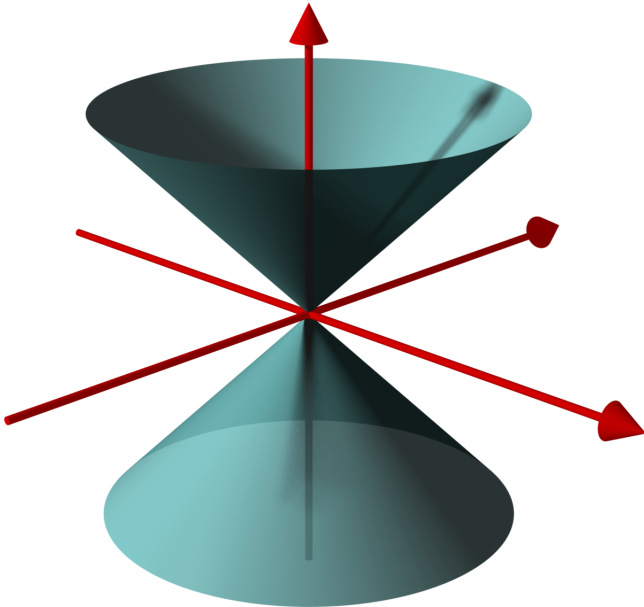
$$y^2 = x^3 - x + 1$$

## Application: Elliptic curve cryptography

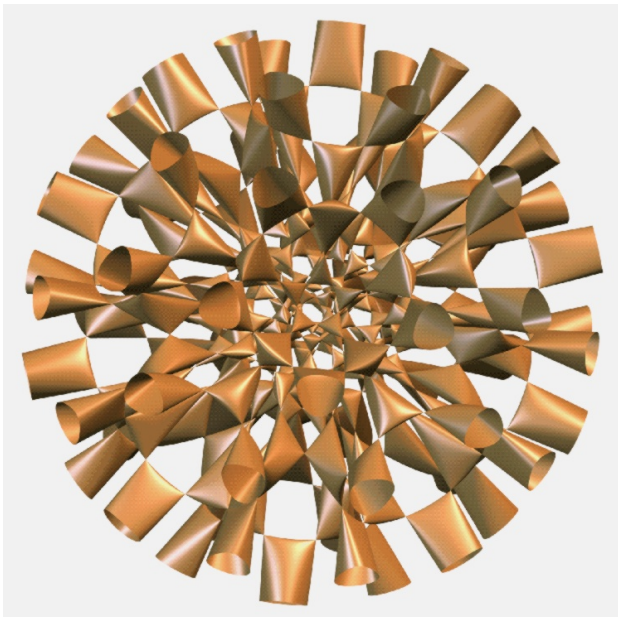


# Singularities

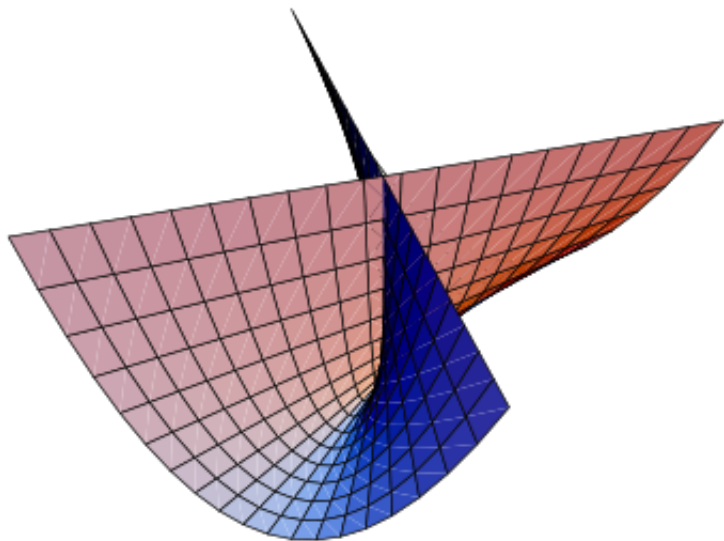
# Singularities



## a Sarti surface



## the Whitney umbrella



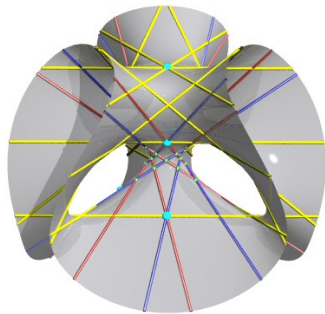
## Singularities from equations

$$x^2y + zy^2 + z^2 + x = 0$$

$$x^2 + y^3 + z^3 + xyz = 0$$

## Singularities from equations

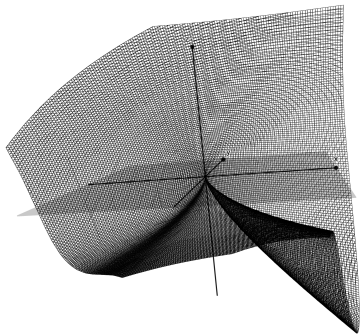
$$x^2y + zy^2 + z^2 + x = 0$$



Clebsch cubic

AE

$$x^2 + y^3 + z^3 + xyz = 0$$



D4 surface singularity



## Some of my research

- Study singularities via methods from other parts of mathematics - homological algebra, representation theory, homotopy theory

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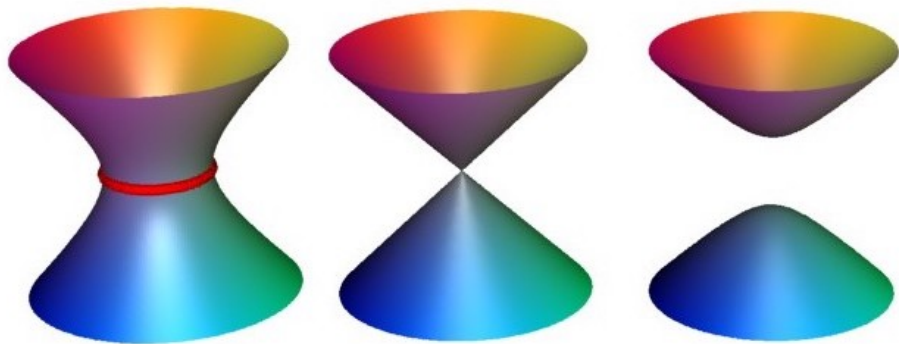
- Study singularities via methods from other parts of mathematics - homological algebra, representation theory, homotopy theory
- Classification of Calabi–Yau threefolds via singularities

## Some of my research

- Study singularities via methods from other parts of mathematics - homological algebra, representation theory, homotopy theory
- Classification of Calabi–Yau threefolds via singularities
- Interactions with deformation theory

## Deformation theory

Study **smoothings** like



Thanks for listening!