Errata to the published version of Singularity categories via the derived quotient

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After the publication of the above paper, Hongxing Chen and Zhengfang Wang informed me that there was a problem with the identification of the kernel of Σ in Proposition 6.2.6. This has some consequences for the rest of the paper. The errors are mainly in section 6.2, 8.1, and 8.2. The main theorems of the paper remain true.

In this document I list the errata to the published version. A corrected version of the paper appears

- On the ArXiv at https://arxiv.org/abs/2003.05439
- On my website at http://mattbooth.info/papers/singcats.pdf

and in this document I also list the changes made in these versions.

Errata Theorem numbering here refers to the published version.

- 1. The identification of the kernel of Σ in 6.2.6 is not correct. The fault in the proof is in asserting that the two ways of getting from $\operatorname{per}_{\mathrm{fg}}(A/\mathbb{L}AeA)$ to $D_{\mathrm{sg}}(R)$ agree.
- 2. In 6.2.9, one needs to change "**per**_{fg} $(A/\mathbb{L}AeA)$ " to "ker Σ ". It is also unclear whether or not $\bar{\Sigma}$ is fully faithful.
- 3. The proof of 6.2.12 needs a new argument. In the current version I deduce this from the more careful analysis of the kernel of Σ in section 6.5; although it appears later one could (and indeed should) carry out the proofs in 6.5 as soon as one has defined Σ .
- 4. 6.2.14 is no longer interesting. 6.2.15 is likely incorrect as stated; it ought to be possible to use the extant proof to give a 'correct' version. 6.2.16, 6.2.17, and 6.2.18 consequently require some changes.
- 5. In 6.3.2, one needs to change " $\mathbf{per}_{\mathrm{fg}}^{\mathrm{dg}}(A/^{\mathbb{L}}AeA)$ " to an appropriate dg enhancement of ker Σ . The proof also requires some minor changes.

- 6. Section 8.1 up to and including 8.1.7 is correct, but perhaps not so useful. 8.1.8 and 8.1.9 are incorrect as written, but they remain true under some smoothness assumptions (precisely, the ones from 6.2.12).
- 7. 8.1.10 is correct but requires a different proof; one can deduce it from a general result about dg categories. The old proof still works in the smooth case.
- 8. The result of section 8.2 is only true under some smoothness assumptions (again, the ones from 6.2.12).

Changes made I tried to ensure that the theorem numbering in the new document reflects the published version as closely as possible. This means that the new version has some extraneous parts, mostly in section 8.

- 1. New section at the end of the introduction.
- 2. Proof of Lemma 6.2.2 improved slightly, as I need to use the argument again in the new section 6.5.
- 3. Identification of the kernel in 6.2.6 has been dropped.
- 4. 6.2.9 corrected. New sentence afterwards about the kernel of the projection $\mathcal{C} \to \mathcal{C}/\mathcal{D}$ when \mathcal{D} is not thick in \mathcal{C} .
- 5. Proof of 6.2.12 corrected.
- 6. 6.2.14 through 6.2.18 were removed.
- $7.\ 6.3.2$ corrected.
- 8. There is a new section 6.5, which gives a partial identification of the kernel of the singularity functor Σ . Logically, this should appear as part of section 6.2, but this would necessitate changes in the extant theorem numberings.
- 9. 8.1.8 and 8.1.9 were folded together, and the smoothness assumptions were added.
- 10. 8.1.9 was replaced with the result that one needs to use to prove 8.1.10. This almost previously appeared in the proof of 8.2.1.
- 11. Proof of 8.1.10 corrected.
- 12. Section 8.2 was expanded a little. The required smoothness assumptions were added to the statement of 8.2.1.